

THE DYNAMICAL ORIGIN OF MULTIPLE STELLAR POPULATIONS IN GLOBULAR CLUSTERS

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ABSTRACT

For a long time globular clusters (GCs) have been considered as ideal laboratories to study dense stellar systems. However, recent observations have revealed that most of them consist of different stellar populations. In particular, omega Centauri (ω Cen) is one of the most complex GCs and has attracted a lot of attention, because its high mass may depict a link between GCs and larger stellar systems, such as ultra compact dwarf galaxies (UCDs). Here we show with direct-summation N -body simulations that a natural explanation is the merger of GCs in colliding galaxies, such as the Antennae galaxy. Hubble Space Telescope observations of these interacting galaxies show clusters of clusters, regions in the galaxy that span hundreds of parsecs, where many of the GCs are doomed to collide. Also, observations of the Large Magellanic Clouds (LMC) reveal that a fraction of GCs are bound and thus will result in a merger. Our results prove that colliding GCs with different metallicities and ages can produce a GC with multiplicity and occupation fractions similar as observed. In our scenario the merged clusters have a phase with a larger amount of flattening than average, and we find that observational data correlates evidence of rotation with multiplicity, which corroborates our hypothesis. Future observations of multiple populations should focus on GCs with ellipticity above average.

1. MOTIVATION

Thanks to the Hubble Space Telescope (HST) we have a growing number of observations that contradict the traditional narrative about GCs being perfect laboratories for studying the structure and evolution of stars; i.e. simple stellar populations that are coeval and chemically homogeneous. Independent observations corroborate that a number of GCs harbor stars which have formed from material from different generation of stars (see for a review Piotto 2009, and references therein). The first discovery of multiple main sequences (MS), ω Cen (Bedin et al. 2004), turned out to be one of the most complex ones, containing not only two but a third, redder MS (Villanova et al. 2007) with the blue MS with twice the metal abundance of the MS dominant red branch (Piotto et al. 2005). Other cases followed; the CMD of NGC 2808 showed a trimodal MS (Carretta et al. 2006; Piotto et al. 2007), NGC 1851 possesses at least two dissimilar stellar populations (Milone et al. 2008) which manifest at the splitting of the sub-giant branch (SGB) in the CMD. In the case of Galactic GCs, we know of many other clusters with a bimodal SGB, as e.g. M22 (Milone et al. 2011), NGC 6388 and M54 (Piotto 2009). In the case of the Large Magellanic Cloud (LMC) up to 70% of the GCs show multiplicity (Milone et al. 2009a).

On the other hand, the merger of stellar clusters is a plausible situation in nature. Observations of colliding galaxies such as the Antennae galaxy show bound systems of young, massive clusters. In this system HST observations exhibit relatively small regions spanning a few hundreds of pc embracing hundreds or even thousands of young clusters, i.e. clusters of clusters or “cluster complexes” (CC from now onwards, see e.g. Kroupa 1998; Whitmore et al. 2010, and references therein). These have been proposed to be the progenitors of UCDs as a result of the agglomeration of hundreds of

their member clusters (Fellhauer & Kroupa 2002, 2005; Brüns et al. 2011; Amaro-Seoane et al. 2011). Also, in the Magellanic Clouds (MCs) a large fraction of clusters are observed in bound binaries or low-order multiples. Dieball et al. (2002) estimate that about 1/8 of the clusters in the Large Magellanic Cloud is a member of a bound group.

In this Letter we show that CCs and the LMC are a natural breeding ground for multiple-populations GCs because they collide and merge. We run direct-summation N -body simulations and analyse the impact of dynamics on the occupation fractions of the different populations in the CMDs. We show that two colliding GCs produce a system with phases of ellipticity significantly over average and, as a matter of fact, observational data reveal that GCs harboring multiple stellar populations are likely to have a larger amount of rotation or ellipticity than average values, so that future surveys of multiple stellar populations should focus on these.

2. SMASHING CLUSTERS

We set initially the clusters on a parabolic orbit so that the minimum distance at which they pass by is d_{\min} if they are considered to be point particles at their centers of mass, as described in Amaro-Seoane (2006). We integrate the evolution with direct-summation N -body tools, which integrate all gravitational forces for all particles at every time step, without making any a priori assumptions about the system. The codes we have employed, N -body and Myriad use the improved Hermite integration scheme (Aarseth 1999, 2003; Konstantinidis & Kokkotas 2010). This needs computation of not only the accelerations, but also their time derivatives. The programmes also include *KS regularisation* and *chain regularisation*, so that when particles are tightly bound or their separation becomes too small during a hyperbolic encounter, the system is regularised (Kustaanheimo & Stiefel 1965; Aarseth 2003) to prevent too small individual time steps. Direct-summation N -body codes scale as N_{\star}^2 , or $\Delta t \propto t_{\text{dyn}}$, with t_{dyn} the dynamical time. For this reason we have employed a version of Myriad which runs on the GRAPE, and the GPU version of N -body of Sverre Aarseth (Aarseth 2003; Konstan-

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		Parameters			
		W_0	z	age (Myr)	Distances
Case \mathcal{A}	Cluster 1	9	0.002	100	$R_{\text{COM}} = 12$
	Cluster 2	5	0.001	50	$d_{\text{min}} = 0.5$
Case \mathcal{B}	Cluster 1	12	0.006	50	$R_{\text{COM}} = 10.6$
	Cluster 2	5	0.005	100	$d_{\text{min}} = 0.5$
Case \mathcal{C}	Cluster 1	12	0.02	50	$R_{\text{COM}} = 10.6$
	Cluster 2	5	0.01	100	$d_{\text{min}} = 0.5$
Case \mathcal{D}	Cluster 1	9	0.002	100	$R_{\text{COM}} = 52.8$
	Cluster 2	5	0.001	10	$d_{\text{min}} = 2.0$
Case \mathcal{E}	Cluster 1	9	0.002	100	1 + 2 from \mathcal{A}
	Cluster 2	5	0.001	50	$R_{\text{COM},2} = 49$
	Cluster 3	5	0.009	2.1×10^3	$d_{\text{min},2} = 10$

TABLE 1

INITIAL CONDITIONS FOR THE CLUSTERS. FROM THE LEFT TO THE RIGHT, WE HAVE THE KING PARAMETER, INITIAL METALLICITY AND THE INITIAL AGE OF THE CLUSTER IN MYRS. ON THE LAST RIGHT COLUMN WE SHOW THE INITIAL DISTANCE BETWEEN THE CENTER-OF-MASS OF THE TWO CLUSTERS THAT COLLIDE, R_{COM} (WHICH CORRESPONDS TO $|\mathbf{x}_1 + \mathbf{x}_2|$ OF FIG. 1 IN AMARO-SEOANE (2006)), AND d_{min} , BOTH IN PC. IN CASE \mathcal{E} WE USE THE OUTCOME OF \mathcal{A} AND MAKE IT COLLIDE WITH A THIRD CLUSTER, OF $N_* = 20,000$ STARS FOR \mathcal{E} . IN THIS SIMULATION WE ADD THE NEW DISTANCE BETWEEN COM, $R_{\text{COM},2}$ AND $d_{\text{min},2}$, FOR THE SECOND COLLISION.

tinidis & Kokkotas 2010).

3. CMDS: MULTIPLE POPULATIONS AND FRACTIONAL OCCUPATION NUMBERS

We ran different cases with different initial conditions, as shown in table 1. The clusters were modelled initially with a King model of different concentrations and metallicities and evolved for different times with the stellar evolution package *sse*, described in Hurley et al. (2000). For the particular case \mathcal{E} , we use the outcome of simulation \mathcal{A} and make it merge with another cluster (numbered 3) on a parabolic orbit with a new $d_{\text{min},2}$ and $R_{\text{COM},2}$, as indicated on the right of the table 1⁴. During the collision we neglect stellar evolution because in all runs it took approximately a few Myrs and the impact of evolving the masses on the global dynamics is negligible. We find in our simulations a significant mass loss after the merger of the clusters that affects the different occupational fraction numbers of the CMDs.

To determine when the clusters merge, we locate the density centers of the two clusters following Casertano & Hut (1985) and that of the merged system. We stop the simulations when these three centers have coincided for a few t_{dyn} . This allows us to study the distribution of stars due to the dynamics of the system, which is important to understand the impact of mass loss in the CMD and occupational fractions in different shells of mass around the density center of the merged system. The CMD we obtain from the simulations correspond to idealised observational conditions. In real observations the measurements are affected by systematic errors. For instance, in the figure 10 of Bellini et al. (2010), which corresponds to the calibration field of ω Cen for the HST WFC3/UVIS with the filters F275W, F814W, the first one has a RMS of ≤ 0.01 in magnitude for well-measured, unsaturated stars. Saturation for F814W starts around the horizontal branch (HB) level and can increase to $\sim 0.05 - 0.08$ magnitude for faint stars in the CMD, while saturated stars in F814W can have errors up to 0.05 in magnitude⁵. We use this as a representative example and introduce an artificial error in our CMD in F606W

of ± 0.01 , and ± 0.05 in F814W by scattering the data with a Gaussian distribution around the real data to show how the populations would appear in an observation of the system. In Fig.(1) we show the CMD of simulation \mathcal{E} . We can see that at the level of the turn-off point (TO) the SGB splits into three branches.

We then calculate the occupation ratio of the different stellar populations, N_1/N_2 (and $N_3/[N_1 + N_2]$ if we had three different populations) for different shells starting at the density center. The results are shown in table 2. From an observational point of view, there is a debate on whether or not there is a significant difference in the distribution of the different populations in GCs with multiplicity. In the case of NGC 1851, Carretta et al. (2010, 2011) observed evidence of a difference in the distribution of the two populations of red giants. According to them, the metal-poor (MP) component seems to be more centrally concentrated than the metal rich (MR) one. This observational evidence contradicts the theory of the formation of the second population of stars within the same cluster (Bekki 2011), in which the metal rich population should be more concentrated around the center. On the other hand, Milone et al. (2009b) did not find any significant radial trend in the number ratio between the two populations of stars of the SGB (although see Carretta et al. 2011). The distribution of the different populations in the final cluster of our models depends mostly on the initial concentration, metallicities, and also on the initial number of stars in each cluster, as well as their initial mass function and ages. In most of our models (all but \mathcal{B} and \mathcal{C}), the metal-rich cluster, which is always cluster 1, appears to have fewer stars in the center of the final cluster than the metal-poor population, which is a natural result of the initial King parameter. ω Cen is the system for which we have the best observational data about the radial distribution of its multiple populations. Bellini et al. (2009) do a detailed study showing that stars belonging to the blue MS (bMS) appear to be more centrally concentrated than stars of the red MS (rMS), with the fraction $N_{\text{bMS}}/N_{\text{rMS}} < 1$ outside the core of the system. Since the bMS contains stars with greater metallicity, according to the authors, stars with greater metallicity appear to be more centrally concentrated. As shown in table 2, one of our models (\mathcal{B}) shows a distribution in which the MR stars appear more centrally concentrated relative to the MP stars of the final cluster. In the same model, metal poor stars appear to dominate the innermost region, while metal rich stars again dominate the external shells of the cluster as a result of the large difference in the initial concentration of the two clusters.

4. RESULTS AND CONCLUSIONS: THE ROLE OF ELLIPTICITY AS A SIGNATURE

In this paper we have addressed the observational evidence of multiplicity of stellar populations in globular clusters in terms of collisions of clusters. Interacting galaxies such as the Antennae or the LMC are natural loci for clusters to collide. In the later, CCs have been observed with the HST and they are the natural birth-place of UCDs, as proposed by a number of works (Kroupa 1998; Fellhauer & Kroupa 2002, 2005; Brüns et al. 2011; Amaro-Seoane et al. 2011).

While dynamics does not affect the shape of the resulting CMD of the merged system, it impinges the number of stars in different parts of it: the occupation fraction will vary because of the dynamics.

After the collision, the resulting cluster has a significant amount of rotation. This depends on the initial conditions

⁴ For movies see, http://www.aei.mpg.de/~pau/Amaro-Seoane_Konstantinidis_2011/index.html

⁵ A. Bellini, private communication

Shell (pc)	N_1/N_2 (and $N_3/(N_1+N_2)$)				
	Case A	Case B	Case C	Case D	Case E
$0 \leq r < 0.5$	0.94	1.25	1.08	0.50	0.85 (0.32)
$0.5 \leq r < 1$	0.86	1.02	1.03	0.53	0.80 (0.26)
$1 \leq r < 2$	0.72	0.83	0.87	0.61	0.78 (0.23)
$2 \leq r < 3$	0.66	0.81	0.82	0.90	0.74 (0.23)
$3 \leq r < 4$	0.82	1.00	0.98	1.07	0.79 (0.24)
$4 \leq r < 5$	1.03	1.06	1.00	1.25	0.77 (0.27)
$5 \leq r < 10$	1.39	1.19	1.17	1.52	0.95 (0.30)
$10 \leq r < 50$	1.96	1.42	1.36	2.28	1.31 (0.45)
$\epsilon_{5, \max}$	0.178	0.215	0.100	0.174	0.298

TABLE 2

OCCUPATION FRACTION AND MAXIMUM AND MAXIMUM ELLIPTICITY FOR THE MASS FRACTION 5, $\epsilon_{5, \max}$, FOR THE CASES OF TABLE 1. THE FRACTIONS ARE GIVEN IN TERMS OF NUMBERS OF STARS BELONGING INITIALLY TO THE FIRST CLUSTER (N_1) AND THE SECOND ONE (N_2) FOR DIFFERENT SHELLS OF THE RESULTING MERGED CLUSTER STARTING FROM THE DENSITY CENTER. FOR THE LAST TWO CASES WE ALSO GIVE THE FRACTION RELATIVE TO THE THIRD ONE, N_3 .

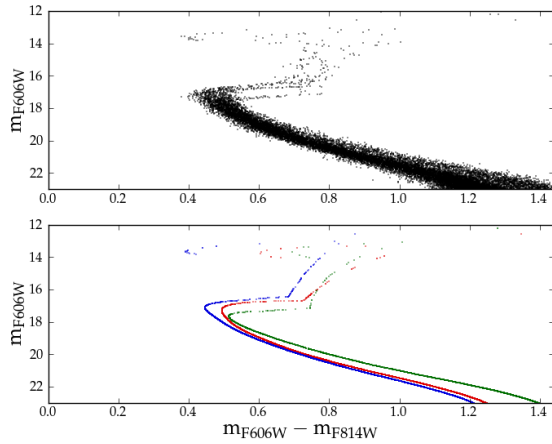


FIG. 1.— CMD of case E, a collision of a cluster with another cluster which is the result of a merger of two clusters itself (case A) with artificial errors (upper panel) and without them (lower panel). The first RMS (curve with the TO point at ~ 0.45 in $m_{F606W} - m_{F814W}$ in the lower panel, blue line in the on-line version of the article) corresponds to the stellar population of $z = 0.002$, the next RMS to $z = 0.001$ (red curve of lower panel) and the third one to $z = 0.009$ (green curve). The clusters that harboured the two first metallicities had an age of 100 and 50 Myrs respectively, and the third one had an age of 10^3 Myrs. The CMD is for stars in the shell $0 \leq r/\text{pc} < 2.5$ after evolving the merged cluster for 9 Gyrs. We set the distance of 4850 pc to convert to apparent magnitudes.

such as the impact parameter, the King parameter and the relative velocity. A comprehensive study of the parameter space and impact on the oblateness of the merged systems as well as the evolution of rotation and CMDs is out of the goals of this work and will be presented elsewhere. Indeed, we have direct measurement of the rotational velocity of a few GCs that corroborates a correlation between rotation and multiplicity of stellar populations. In most of the observations, rotation is differential (Meylan & Mayor 1986). The results for the rotational velocities of 11 GCs are summarized in Table 7.2 of Meylan & Heggie (1997), where the rotational velocities v_{rot} are measured in terms of the velocity dispersion σ of each cluster. In Table 3 we show the ϵ and the multiplicity of a few GCs for which we have data: 9 out of the 11 clusters show evidence of multiplicity. For the other two there are no observational data about multiplicity to our knowledge. The

Cluster	ϵ	V_{rot}/σ	Multiplicity
NGC 5139	0.17	0.32	3MS,6SGB,4RGB (Bellini et al. 2010)
NGC 6656	0.14	0.50	2SGB,2RGB (Milone et al. 2011)
NGC 3201	0.12	0.28	2SGB,2RGB (Kravtsov et al. 2010, 2011)
NGC 7089	0.11	0.34	No data available
NGC 6205	0.11	0.25	3RGB (Carretta et al. 2009)
NGC 6341	0.10	0.30	No data available
NGC 104	0.09	0.26	2SGB,2RGB,broad MS (Anderson et al. 2009)
NGC 6397	0.07	0.11	2RGB (Carretta et al. 2009)
NGC 7078	0.05	0.15	2RGB (Carretta et al. 2009)
NGC 5272	0.04	0.12	2RGB (Carretta et al. 2009)
NGC 362	0.01	0.01	2RGB (Pancino et al. 2010)

TABLE 3

ELLIPTICITY, ROTATIONAL VELOCITY AND MULTIPLICITY WITH REFERENCE OF 11 GALACTIC GCs.

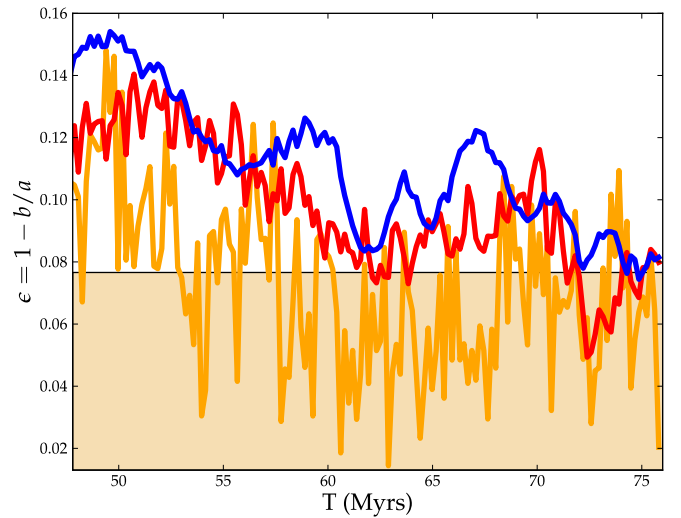


FIG. 2.— Evolution of ϵ for case D after the density centers coincide. The semi-major axes are calculated with the ellipsoids of inertia (see e.g. Chandrasekhar 1969) and determined by different mass fractions of the stars (at $T \sim 47.7$ Myrs, from the bottom to the top 0.2, 0.5 and 0.8, displayed in orange, red and blue, respectively in the on-line article). The solid black line shows $\bar{\epsilon}$; thus, values within the colored box are average. The stars are distributed according to the amount of gravitational energy; hence, the lower the mass fraction is, the closer we are to the centre of the resulted merged system.

fastest rotating cluster is NGC 6656 with $v_{\text{rot}}/\sigma = 0.5$, and an ellipticity of $\epsilon = 0.14$ (with $\epsilon := 1 - b/a$, where a and b are the semi-major and -minor axes). ω Cen rotates very fast, $v_{\text{rot}}/\sigma = 0.32 - 0.41$, which makes it one of the fastest rotating GCs in the Galaxy. This could be a signature for an agglomeration process of a cluster in a CC which receives more and more impacts from other lighter clusters and “runs away” in mass, on its way to forming a UCD (Amaro-Seoane et al. 2011).

In Fig.(2) we show ϵ vs time for D. We start the analysis after the three density centers coincide. The system has an ϵ above average during some few Myrs and by $T \sim 75$ Myrs $\epsilon \sim 0.08$. We have data for ϵ for 100 out of the 157 Galactic GCs (Harris 2010); the average is $\bar{\epsilon} \sim 0.0766$. Some of the clusters for which there is observational evidence of the existence of multiple stellar populations have larger values than $\bar{\epsilon}$. In particular, ω Cen has $\epsilon = 0.17$, which makes this cluster 9th in the list of the most oblate clusters. Other clusters have a rather large value, NGC 2808 ($\epsilon = 0.12$), NGC 6656 ($\epsilon = 0.14$) and NGC 3201 ($\epsilon = 0.12$). 47 Tuc, with multiple

stellar populations (Anderson et al. 2009), has a value slightly above average, 0.09.

We also note that there is no observable difference in the rotation of the multiple populations in GCs. Measurements of the rotation of the 650 stars of ω Cen (Pancino et al. 2007; Anderson & van der Marel 2010) show that all subpopulations rotate as a single one. This can be explained in terms of a collision between GCs, which would assign all stars the same amount of rotation regardless of their population. Also, according to Lane et al. (2010) several of the observed properties of 47 Tuc can be explained if a merger of two clusters occurred in the past. Kravtsov et al. (2011) find that there is a strong segregation between the two populations of stars that exist in NGC 6752 and NGC 3201, with the fainter SGB and redder RGB stars dominating the center of both clusters. This difference in the distribution of the two populations might be the result of a merger between two GCs, as shown by our simulations.

Hence, although this mechanism probably cannot explain all cases observed, in particular the strange case of the metallicities of the different MS of ω Cen, we deem it is a plausible and natural way of creating multiplicity of stellar populations. Also, even if observations on rotation are incomplete and do not cover all of the GCs with multiplicity, we have shown in our limited study of the parameter space that the result of a collision is a cluster that will exhibit phases in the evolution with ϵ above average. In the particular case of simulation \mathcal{D} , the GC achieves average values after a short time, so that any oblateness would not be present today in old clusters, unless

the collision happened recently (though we note that cannot model realistic GCs with our number of stars). This means that GCs, in particular young ones, with ϵ above average are more likely to harbor multiple populations of stars; i.e. any amount of rotation in GCs with multiplicity could be a fingerprint for a dynamical origin. Future observations should focus on such GCs, since these are likely to display multiplicity. In particular, the systems NGC 7089 and NGC 6341 have not been investigated for presence of multiplicity, and in our scenario they are good candidates for having different stellar populations.

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